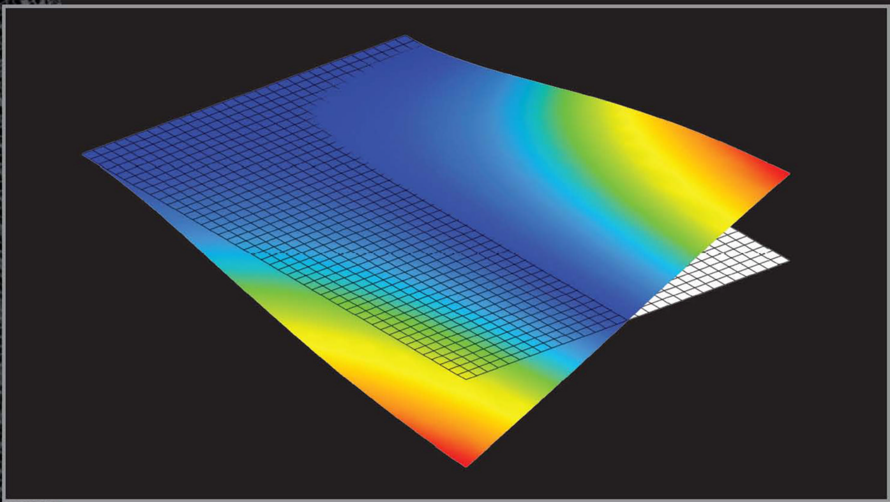
Mechanical Vibrations

Theory and Application to Structural Dynamics

Third Edition



Michel Géradin

Daniel J. Rixen

WILEY

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THEORY AND APPLICATION TO STRUCTURAL DYNAMICS

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Foreword

The first two editions of this book had seven skillfully written chapters, organized in my mind in three parts. Collectively, they aimed at giving the reader a coherent presentation of the theory of vibrations and associated computational methods, in the context of structural analysis. The first part covered the analytical dynamics of discrete systems, and both undamped and damped vibrations of multiple-degree-of-freedom systems. It also served as a good introduction to the second part, which consisted of two chapters. The first one focused on the dynamics of continuous systems and covered the subject of wave propagation in elastic media. It was followed by a chapter which bridged this topic with the first part of the book, by introducing the novice to the concept of displacement methods for semi-discretizing continuous systems. It also culminated with a brief and yet well-executed initiation to the finite element method. All this led to the third part of the book, which indulged into a concise and effective treatment of classical numerical methods for the solution of vibration problems in both frequency and time domains. Covering all of these topics in a unified approach, making them interesting to both students and practitioners, including occasional references to experimental settings wherever appropriate, and delivering all this in less than 400 pages, was a daunting challenge that the authors had brilliantly met. For this reason, the previous editions of this book have been my favourite educational publication on this subject matter. I have used them to teach this topic at the MS level, first at the University of Colorado at Boulder, then at Stanford University.

So what can one expect from a third edition of this book?

In its third edition, the overall organization of this book and that of its chapters has remained mostly unchanged. However, several enhancements have been made to its technical content. The notion of the response of a system to a given input has been refined throughout the text, and its connections to the concepts of dynamic reduction and substructuring (which remain timely) have been made easier to observe, follow, and understand. Chapter 3 has gained a new section on experimental methods for modal analysis and some associated essentials in signal processing and system identification. The mathematical content of Chapter 6 has been somehow refreshed, and its scope has been enhanced by two welcome enrichments. The first one is a new section on linear equation solvers with particular emphasis on singular systems. Such systems arise not only in many mechanical and aerospace engineering problems where the structure of interest is only partially restrained or even unrestrained, but also as artifacts of many modern computational methods for structural analysis and structural dynamics. The second enrichment brought to Chapter 6 is an updated section on the analysis of the sensitivity of frequencies and mode shapes to parameters of interest, and its association with model updating. Most importantly, the third edition comes now with carefully designed problem sets (and occasionally some solutions) that will certainly enhance both processes of teaching and learning. Overall, the third edition has added about 150 pages of technical content that make it a better textbook for students and teachers, a useful reference for practitioners, and a source of inspiration for researchers.

Charbel Farhat
Stanford University
1 January 2014

Preface

This monograph results from a complete recasting of a book on Mechanical Vibrations, initially written in French and published by Masson Éditions in 1992 under the title *Théorie des vibrations, Application à la dynamique des structures*. The first edition in English was issued shortly after, thanks to the support of DIST (French Ministry of Scientific Research and Space) and published by John Wiley & Sons in 1994. The book was indubitably felt to fill a gap since both editions were a success in France as well as internationally, so that both versions were almost immediately followed by a second edition by the same publishers: in French in 1996, and in 1997 for the English version. Due to the short delay between editions, only minor changes – essentially corrections – took place between the first and second versions of the manuscript.

The numerous constructive comments received from readers – university colleagues, students and practising engineers – during the following decade convinced both of us that a deep revision of the original manuscript was definitely needed to meet their expectations. Of course there were still remaining errors to be corrected – and the very last one will never be discovered, error-making being a common trait of human beings – and more rigor and accuracy had to be brought here and there in the presentation and discussion of the concepts. But the subject of mechanical vibration has also rapidly evolved, rendering the necessity of the addition of some new important topics. Proposed exercises to help, on the one hand, teachers explain the quintessence of dynamics and, on the other hand, students to assimilate the concepts through examples were also missing.

We were already planning to produce this third edition in French in the early 2000s, but the project could never be achieved due to overwhelming professional duties for both of us. The necessary time could finally be secured from 2010 (partly due to the retirement of the first author). However, priority has now been given to the English language for the writing of this third, entirely new edition since our perception was that the demand for a new, enhanced version comes essentially from the international market. We are indebted to Éditions Dunod for having agreed to release the rights accordingly.

We are thus pleased to present to our former readers a new edition which we hope will meet most of their expectations, and to offer our new readers a book that allows them to discover or improve their knowledge of the fascinating world of mechanical vibration and structural dynamics.

Without naming them explicitly, we express our gratitude to all those who have helped us to make this book a reality. Indeed, we received from many colleagues, friends and relatives much support, which could take various forms, such as a careful and critical reading of some parts, the provision of some examples and figures, appropriate advice whenever needed, personal support and, not the least, the understanding of our loved ones when stealing from them precious time to lead such a project to its very end.

Michel Géradin and Daniel J. Rixen
München
24 January 2014

Introduction

We owe to Lord Rayleigh the formulation of the principles relative to the theory of vibration such as they are applied and taught nowadays. In his remarkable treatise entitled *Theory of Sound* and published in 1877 he introduced the fundamental concept of oscillation of a linear system about an equilibrium configuration and showed the existence of vibration eigenmodes and eigenfrequencies for discrete as well as for continuous systems. His work remains valuable in many ways, even though he was concerned with acoustics rather than with structural mechanics.

Because of their constant aim to minimize the weight of flying structures, the pioneers of aeronautics were the first structural designers who needed to get vibration and structural dynamic problems under control. From the twenties onwards, aeronautical engineers had to admit the importance of the mechanics of vibration for predicting the aeroelastic behaviour of aircraft. Since then, the theory of vibration has become a significant subject in aeronautical studies. During the next forty years, they had to limit the scope of their analysis and apply methods that could be handled by the available computational means: the structural models used were either analytical or resulted from a description of the structure in terms of a small number of degrees of freedom by application of transfer or Rayleigh-Ritz techniques.

The appearance and the progressive popularization of computing hardware since 1960 have led to a reconsideration of the entire field of analysis methods for structural dynamics: the traditional methods have been replaced by matrix ones arising from the discretization of variational expressions. In particular, the tremendous advances in the finite element method for setting up structural models gave rise to the development of new computational methods to allow design engineers to cope with always increasing problem sizes.

Today, the elaboration of efficient computational models for the analysis of the dynamic behaviour of structures has become a routine task. To give an example, Figure 1 illustrates the computational prediction of the vibration modes of a stator section of an aircraft engine. The fineness of the finite element model has been adapted in this case for the needs of the associated stress analysis, the latter requiring a level of detail that is not really needed for a modal analysis. The eigenmode represented is a 3-diameter mode exhibiting a global deformation of the structure. What makes the modal analysis of such a structure very difficult is the high level

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Figure 1 Finite element model of a stator section of aircraft engine. Source: Reproduced with permission from Techspace Aero – SAFRAN Group.

of cyclic symmetry (resulting from the number of stator blades) which is responsible for the appearance of a high number of nearly equal eigenvalues.

Development of computing, acquisition and sensing hardware has led to a similar revolution in the field of experimental techniques for identification of vibrational characteristics of structures. For more than thirty years, experimental modal analysis techniques have been developed which are based either on force appropriation or on arbitrary excitation.

The methods for dynamic analysis, whether they are numerical or experimental, have now taken an important place everywhere in engineering. If they were rapidly accepted in disciplines such as civil engineering, mechanical design, nuclear engineering and automotive production where they are obviously needed, they have now become equally important in the design of any manufactured good, from the micro-electromechanical device to the large wind turbine.

From its origin in the early sixties, the aerospace department of the University of Liège (Belgium) has specialized mainly in structural mechanics in its education programme. This book results from more than twenty years of lecturing on the theory of vibration to the students of this branch. It is also based on experience gathered within the University of Liège's Laboratory for Aerospace Techniques in the development of computational algorithms designed for the dynamic analysis of structures by the finite element method and implemented in the structural analysis code the team of the laboratory has developed since 1965, the SAMCEF™ software.[1]

The content of the book is based on the lecture notes developed over the years by the first author and later formatted and augmented by one of his former students (the second author). This work reflects the teaching and research experience of both authors. In addition to his academic activity at the University of Liège, the first author has also spent several years as head of the European Laboratory for Safety Assessment at the Joint Research Centre in Ispra (Italy). The second author has accumulated until 2012 lecturing and research experience at

[1] From 1986, SAMCEF™ has been industrialized, maintained and distributed by SAMTECH SA, a spin-off company of the University of Liège.

the Delft Technical University (The Netherlands) and is currently pursuing his career at the Technische Universität München (Germany). The book has been adopted internationally as course reference in several universities.

Due to its very objective, the book has a slightly hybrid character: the concepts of vibration theory are presented mainly with the intention of applying them to dynamic analysis of structures and significant attention is paid to the corresponding methods. Even though the foundations of analytical mechanics are reviewed, a preliminary acquaintance with this subject is necessary. A good knowledge of matrix algebra and theory of complex numbers, calculus, structural mechanics and numerical analysis for linear systems is required. It is also assumed that the reader is familiar with the theory of the single-degree-of-freedom oscillator. However, the presentation of the finite element method is deliberately made simple since its study requires a course of its own. Finally, the very important fields of nonlinear vibration and random vibration have been intentionally omitted in the present text since they are highly specialized subjects.

What is new in this third edition?

Although the overall structure of the book, its organization into individual chapters and the main topics addressed, remain unchanged, this new version is the result of important revision work to achieve major improvements.

Regarding the theoretical content itself, the main changes with respect to the previous edition are the following:

- The response of an either discrete or continuous system has been the object of deep rethinking and turned out to be a thread towards the important concepts of dynamic reduction and substructuring. The latter are explained and developed, starting from the observation that the response of a part from the overall system is the result either from an excitation of its support, or from the application of a set of loads at selected points of the structure. Such duality can be exploited in at least two ways. On the one hand, it allows a system description in terms of the classical concepts of mechanical impedance or admittance. On the other hand, it naturally leads to the concept of dynamic substructuring based on an expansion of the response in terms of the spectral content of the impedance and admittance relationships.
- Experimental modal analysis is an essential ingredient in structural dynamics since it allows to confirm by experiment the structural properties predicted through numerical modelling. Therefore it was felt necessary to include in this new version of the book the essentials of signal processing and identification techniques that allow us to extract the spectral properties of a linear structure from measured dynamic responses.
- In the same spirit, the concept of eigensolution sensitivity to physical parameters has been further detailed since it is the basis for the development of appropriate numerical tools for improving the numerical model of a real dynamic system.
- The considerable evolution of the size of the structural systems to be considered for eigenvalue extraction and transient dynamic analysis in the context of large engineering projects had to be reflected and addressed properly. Models reaching the size of several millions of degrees of freedom (such as the one displayed on Figure 1) are now common practice. The efficiency of the eigenvalue solvers (such as the Lanczos method) and implicit

time integrators (based on the Newmark family) depends for one part on the tuning of the algorithms themselves, but perhaps even more on the performance of the linear solvers that are used at each solution step. Therefore it was felt necessary to cover in a deeper manner the topic of linear solvers, introducing not only the principle of the algorithms but also their implementation taking into account the sparse character of the large sets of equations generated by finite element discretization. Much attention is also brought to the case of singular systems since they frequently occur in the context of structural dynamics.

– The link that was made in the previous edition between vibration and wave propagation did not allow the reader to easily grasp the physical nature of the wave propagation phenomena that can occur in a continuous medium. The discussion of the fundamental cases of wave propagation in solids (both in one-dimensional and three-dimensional media) has thus been reviewed and better illustrated in order to improve the didactics of the presentation.
– The presentation of the finite element method has still been limited to one-dimensional structures (bars, beams) since the main objective of the book is not to go deeply into finite element technology. The chapter devoted to it has, however, been complemented with the development of a beam element including the shear deformation. The motivation behind the presentation was to show that, as is often the case, remaining within the strict context of the variational principle of displacements leads to shortcomings which can easily be removed through the use of mixed variational formulations.
– The stability and accuracy properties of the Newmark family of time integration algorithms have been revisited, their rigorous discussion being achieved in terms of the invariants of the amplification matrix. Also, it is shown that dissociating displacement interpolation and expression of equilibrium allows us to imbed most integration schemes of the Newmark family in the same formalism.

As a result, the original manuscript has been almost completely rewritten. The opportunity has been taken to improve or clarify the presentation whenever necessary, including the quality of the figures.

A great effort has been achieved to adopt throughout the manuscript notations that are as coherent and uniform as possible. Therefore a general list of notations and symbols is provided after this introduction. However it was still necessary in many cases to depart locally from these general conventions, and therefore to introduce in each chapter an additional list of local definitions that complements the general one.

Among the many constructive comments received regarding the previous editions, a major deficiency felt and reported by the users was the absence of exercises proposed to the reader. A few solved exercises are now detailed at the end of each chapter, and both teachers and students will certainly appreciate the fact that a number of selected problems are also suggested. The numerical solution of some of them requires the use of a numerical toolbox, in which case softwares such as MATLAB® or the Open Source ones OCTAVE® and SCILAB® are appropriate. Some others involve cumbersome analytical developments that are greatly facilitated by symbolic computation using software tools such as MAPLE® or MATHEMATICA®.

The content

The content of the book is organized as follows:

Chapter 1 is dedicated to analytical dynamics of discrete systems. Hamilton's principle is taken as a starting point: first the equations of motion are found for one particle and then those for a system of particles under kinematic constraints are derived. Considering the equations of motion in the Lagrangian form, the structure of the inertia terms and the classification of the forces are established. In the last two sections of the chapter, the less common case of impulsive loading of systems is dealt with and the method of Lagrange multipliers is introduced.

Chapter 2 discusses the undamped vibrations of n-degree-of-freedom systems and begins by introducing the concepts of equilibrium position and of equilibrium configuration corresponding to steady motion. After a review of the classical concepts of eigenmodes and eigenfrequencies, some more specific aspects are considered: the forced harmonic response is developed and is shown to lead to the concepts of dynamic influence coefficient and mechanical impedance. The modal expansion technique is applied for calculating the dynamic response to transient external loading. It is shown that limiting the points of load application leads to the concept of reduced mechanical admittance. The case of systems excited through support motion is discussed in depth and also examined from the point of view of dynamic substructuring. Variational methods for characterizing the eigenvalues of a vibrating system are then discussed. The solution of the motion equations of rotating systems is considered in a specific section at the end of the chapter, with the main objective to show the existence of instability zones linked to the existence of gyroscopic forces.

Chapter 3 deals with the damped oscillations of n-degree-of-freedom systems. First, the concept of lightly-damped systems and its equivalence to the modal damping assumption are discussed. Then the principles of modal identification through appropriate excitation and the characteristic phase-lag theory are outlined. The formulation of damped system equations in state-space form is developed in order to provide a suitable mathematical model for describing systems with arbitrarily large damping. In the last section, a basic presentation is made of the signal processing and identification techniques that are commonly used to best fit the parameters of the mathematical model from experimental measurements.

In Chapter 4, the theory of vibration is extended to the analysis of continuous systems, taking as a starting point the variational principle operating on displacements. The chapter begins by considering the case of three-dimensional continuous media: strain measure, stress–strain relationships, variational formulation and equations of motion. The effects of the second-order terms arising from the presence of an initial stress field are investigated in detail. Then the concepts of eigenmodes, eigenfrequencies and modal expansion are generalized to the continuous case. It is also shown that the principle of reciprocity commonly described for structures under steady loading can be generalized to dynamics. In a major part of the chapter, a quite extensive study is made of some one-dimensional or two-dimensional continuous systems: the bar in extension, the vibrating string, the bending of a beam without and with shear deflection and finally the bending vibration of thin plates. Numerous examples of closed-form solutions are given and particular attention is devoted to the effects resulting from the rotation of beams and, for systems in bending, from initial extension. Their respective properties as one-dimensional wave guides are discussed. The last section provides an elementary presentation of wave propagation phenomena in an elastic medium, with a derivation of the fundamental solutions and a discussion of their physical meaning.

In Chapter 5, the approximation problem for continuous systems is investigated by means of displacement methods. First, the Rayleigh–Ritz method is reviewed and then applied to some classical problems such as the bar in extension, and the bending of a beam and of a thin plate. The case of prestressed structures is once more considered. The second part of the chapter is dedicated to an introduction to the finite element method, the principles of which are illustrated by several simple examples. The chapter ends with the more complex but instructive case of finite element modelling of the beam with shear deformation.

Chapter 6 deals with solution methods for the eigenvalue problem. After an introduction where a classification of the existing methods is suggested, a successive survey of the most classical methods is made and their related numerical aspects are discussed. The methods efficiently implemented in structural computation codes are pointed out, namely the subspace method and the Lanczos algorithm. A significant part of the chapter is devoted to the efficient solution of large, sparse linear systems since they form in fact the kernel of eigenvolvers based on inverse iteration such as Lanczos and subspace iteration. The particular case of singular structures is discussed in depth since frequently occurring in the context of structural dynamics. The methods of dynamic reduction and substructuring already introduced in Chapter 2 are discussed again, with three objectives in mind: to review the principle of dynamic reduction in a more general manner, to show that dual points of view can be adopted, depending upon the physical nature (displacement or force) of the primary variables and to propose dynamic reduction and substructuring as a practical approach for the solution of large problems of structural dynamics. A section is also devoted to the computation of error bounds to eigenvalues. The last section deals with the concept of eigensolution sensitivity to structural modifications.

Chapter 7 outlines some aspects of direct methods for integrating the transient dynamic response. After having introduced the concepts of stability and accuracy for an integration operator, it discusses the one-step formulas of Newmark's family. Their properties are analyzed as well as those of variants commonly used in structural analysis: the Hilber-Hugues-Taylor α-method and the Generalized-α variant which provides a neat way to introduce numerical damping in the model and the central difference integration scheme especially well adapted to impact problems. Eventually, there is a short discussion of the time integration of nonlinear systems.

The scope

The book has been devised to be used by senior undergraduate and graduate students. Therefore, the associated concepts are revealed by numerous simple examples. Nevertheless, although the text is primarily aimed at students, it is also dedicated to research and design engineers who wish to improve their understanding and knowledge of the dynamic analysis of structures. Solved exercises are also proposed to readers at the end of each chapter, and a number of selected problems are provided to allow them to practice the concepts and assess their assimilation. In order to simplify the presentation, most examples and solved exercises are presented in a nondimensional manner.

Finally, the authors do not claim to cover within the following text the field of vibration theory and dynamic analysis in an exhaustive way. Neither have they made explicit reference to all the bibliographic work they have consulted throughout their writing. Therefore the following list of references is suggested for further details on the various aspects of structural dynamics.

Suggested bibliography

Reference works on linear and nonlinear vibration theory (phenomenological aspects and analytical methods):

Biezeno CB and Grammel R 1939 *Technische Dynamik*. Springer Berlin.

Crede CE and Harris CM 1977 *Shock and Vibration Handbook* (2nd edition). McGraw-Hill, New York.

Del Pedro M and Pahud P 1997 *Mécanique vibratoire: systèmes discrets linéaires*. Presses Polytechniques Romandes, Lausanne.

Den Hartog J 1947 *Mechanical Vibrations*. McGraw-Hill, New York.

Genta G 2009 *Vibration Dynamics and Control*. Springer.

Géradin M 1990 *Dynamique des constructions mécaniques*. Course lecture notes, University of Liège.

Hagedorn P 1981 *Non-linear Oscillations*. Clarendon Press, Oxford.

Hussey M 1983 *Fundamentals of Mechanical Vibrations*. Macmillan.

Lalanne M, Berthier P and Der Hagopian J 1980 *Mécanique des vibrations linéaires*. Masson, Paris.

Lord Rayleigh BJWS 1894 *Theory of Sound (2 volumes)* 2nd edn. Macmillan and Co., London and New York (first edition in 1877).

Magnus K 1961 *Schwingungen; eine Einfuhrung in die theoretische Behandlung von Schwingungsproblemen*. Teubner, Stuttgart.

Mazet R 1966 *Mécanique vibratoire*. Dunod, Paris.

Meirovitch L 1967 *Analytical Methods in Vibrations*. The Macmillan Company, New York.

Meirovitch L 1975 *Elements of vibration analysis*. McGraw-Hill.

Müller PC and Schiehlen WO 1985 *Linear Vibrations: A Theoretical Treatment of Multi-Degree-of-Freedom Vibrating Systems*. Martinus Nijhoff Publishers, Dordrecht.

Nayfeh AH and Mook DT 1979 *Nonlinear Oscillations*. John Wiley & Sons, Inc., New York.

Rao S 1986 *Mechanical Vibrations*. Addison-Wesley, Reading, MA.

Rocard Y 1960 *Dynamique générale des vibrations*. Masson, Paris.

Roseau M 1984 *Vibrations des systèmes mécaniques: méthodes analytiques et applications*. Masson, Paris.

Timoshenko S 1937 *Vibration Problems in Engineering* 2nd. edn. D. Van Norstrand.

Tong KN 1960 *Theory of Mechanical Vibration*. John Wiley & Sons, Inc., New York.

Reference works on random vibration:

Crandall SH and Mark WD 1963 *Random Vibration in Mechanical Systems*. Academic Press, New York.

Preumont A 1994 *Random Vibrations and Spectral Analysis*. Kluwer Academic.

Reference works on structural dynamics:

Argyris J and Mlejnek H 1991 *Dynamics of Structures*. Elsevier, Amsterdam.

Clough RW and Penzien J 1975 *Dynamics of Structures*. McGraw-Hill, New York.

Craig R and Kurdila A 2006 *Fundamentals of Structural Dynamics*. John Wiley & Sons, Inc., New York.

Fraeijs de Veubeke B, Géradin M and Huck A 1972 *Structural Dynamics*. number 126 in *CISM Lecture Notes (Udine, Italy)*. Springer.

Hurty WC and Rubinstein MF n.d. *Dynamics of Structures*. Prentice-Hall New Jersey.

Meirovitch L 1980 *Computational Methods in Structural Dynamics*. Sijthoff & Noordhoff.

Preumont A 2013 *Twelve Lectures on Structural Dynamics*. Springer.

Reference works dedicated to the finite element method:

Bathe K 1996 *Finite Element Procedures in Engineering Analysis*. Prentice Hall, Englewood Cliffs NJ.
Batoz JL and Dhatt G 1990 *Modélisation des structures par éléments finis* vol. 1, 2 and 3. Presses Université Laval.
Craveur JC 1996 *Modélisation des structures, calcul par éléments finis*. Masson, Paris.
Dhatt G and Touzot G 1984 *Une présentation de la méthode des éléments finis*. Collection de l'Université de Compiègne, edited by Maloine.
Hughes T 1987 *The Finite Element Method – Linear Static and Dynamic Finite Element Analysis*. Prentice Hall Inc., New Jersey.
Imbert J n.d. *Analyse des structures par éléments finis*. Cepadues Editions, Toulouse.
Kardestuncer H and Norrie DH 1987 *Finite Element Handbook*. McGraw-Hill, Inc.
Zienkiewicz O and Taylor R 1989 *The Finite Element Method* 4th edn. McGraw-Hill. 2 volumes.

Reference works on aeroelasticity:

Bisplinghoff RL and Ashley H 1975 *Principles of Aeroelasticity*. Dover Publications, New York.
Bisplinghoff RL, Ashley H and Halfman R 1955 *Aeroelasticity*. Addison-Wesley, Cambridge, Mass.
Dowell E, Clark R, Cox D, Curtiss H, Edwards J, Hall K, Peters D, Scanlan R, Simiu E and Sisto F 2004 *A Modern Course in Aeroelasticity (Fourth Revised and Enlarged Edition)*. Kluwer Academic Publishers.
Förschung HW 1974 *Grundlagen der Aeroelastik*. Springer-Verlag, Berlin.
Fung Y 1955 *An Introduction to the Theory of Aeroelasticity*. John Wiley & Sons, Ltd, Chichester.

Reference works on fluid-structure interaction and acoustics:

Morand-Fahy FJ and Gardonio P 2007 *Sound and structural vibration:radiation, transmission and response*. Academic press.
Morand HJP and Ohayon R 1992 *Interactions fluides-structures*. Masson, Paris.
Ohayon R and Soize C 1997 *Structural Acoustics and Vibration: Mechanical Models, Variational Formulations and Discretization*. Elsevier.

Reference works on classical mechanics:

Goldstein H 1986 *Classical Mechanics*. Addison-Wesley, Reading, MA.
Komzsik L 2014 *Applied Calculus of Variations for Engineers*. 2nd edn. CRC Press.
Lanczos C 1949 *The Variational Principles of Mechanics* Mathematical Expositions, No. 4. University of Toronto Press, Toronto, Canada.
Meirovitch L 1970 *Methods of Analytical Dynamics*. McGraw Hill, New York.
Whittaker ET 1952 *Analytical Dynamics of Particles and Rigid Bodies*. Cambridge University Press, New York.

Reference works on solid mechanics:

Fraeijs de Veubeke B 1979 A Course in Elasticity vol. 29 of *Applied Mathematical Sciences* Springer.
Fung Y 1965 *Foundations of Solid Mechanics*. Prentice Hall, New Jersey.

Reference works on dynamics of mechanical systems:

Bauchau OA 2011 *Flexible Multibody Dynamics*. vol. 176. Springer Science.
Géradin M and Cardona A 2001 *Flexible Mutibody Dynamics: The Finite Element Method Approach*. John Wiley & Sons, Ltd, Chichester.
Pfeiffer F 2005 *Mechanical System Dynamics* vol. 40. Springer.
Shabana AA 2005 *Dynamics of Multibody Systems*. Cambridge University Press.

Reference works on experimental methods in vibration:

Brandt A 2011 *Noise and Vibration Analysis: Signal Analysis and Experimental Procedures*. John Wiley & Sons, Ltd, Chichester.

Brincker R 2013 *Introduction to Operational Modal Analysis*. John Wiley & Sons, Ltd, Chichester.
Ewins DJ 2000 *Modal Testing: Theory, Practice and Application*. Research Studies Press, Baldock.
Heylen W, Lammens S and Sas P 1997 *Modal Analysis Theory and Testing*. Katholieke Universiteit Leuven, Department Werktuigkunde, Heverlee.
McConnell KG and Varoto PS 2008 *Vibration Testing: Theory and Practice* second edn. John Wiley & Sons, Ltd, Chichester.
Silva JM and Maia NM 1999 *Modal analysis and testing NATO Science Series E* vol. 363 Springer.
Zaveri K 1984 *Modal Analysis of Large Structures – Multiple Exciter Systems*. Brüel & Kjær, BT 0001-11.

Reference works on vibration control:

Preumont A 2006 *Mechatronics: Dynamics of Electromechanical and Piezoelectric Systems* vol. 136. Springer.
Preumont A 2011 *Vibration Control of Active Structures: An Introduction* vol. 179. Springer.

Reference works on rotor dynamics:

Friswell MI, Penny JE, Garvey SD and Lees AWi 2010 *Dynamics of Rotating Machines*. Cambridge University Press.
Genta G 2005 *Dynamics of Rotating Systems*. Springer, New York.
Lalanne M, Ferraris G and Der Hagopian J 1998 *Rotordynamics Prediction in Engineering* (2nd edition). John Wiley & Sons, Inc., New York.

Reference works on numerical methods for linear and eigenvalue solutions:

Golub GH and Van Loan CF 1989 *Matrix Computations*. John Hopkins University Press.
Householder AS 1964 *The Theory of Matrices in Numerical Analysis*. Blaisdell.
Komzsik L 2003 *The Lanczos Method: Evolution and Application*. SIAM.
Wilkinson J 1965 *The Algebraic Eigenvalue Problem*. Clarendon Press, Oxford.

Reference works on numerical methods for time integration:

Belytschko T and Hughes T 1983 *Computational Methods for Transient Analysis*. North-Holland.
Gear C 1971 *Numerical Initial Value Problems in Ordinary Differential Equations*. Prentice Hall, Englewood Cliffs, NJ.

List of main symbols and definitions

The list below provides the definitions of variables and quantities that are common to all book chapters. A separate list is provided at the beginning of each chapter with definitions that remain local to the chapter.[2]

In the text, we will use bold characters to denote matrices. Lower case bold symbols will represent uni-column matrices whereas upper case ones denote multi-column matrices. For instance,

$$\boldsymbol{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \end{bmatrix}$$

[2] Multiple use of same symbol has been avoided as much as possible, but may still occur locally.